$f(T)$ gravity mimicking dynamical dark energy. Background and perturbation analysis

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We investigate $f(T)$ cosmology in both the background, as well as in the perturbation level, and we present the general formalism for reconstructing the equivalent one-parameter family of $f(T)$ models for any given dynamical dark energy scenario. Despite the completely indistinguishable background behavior, the perturbations break this degeneracy and the growth histories of all these models differ from one another. As an application we reconstruct the $f(T)$ equivalent for quintessence, and we show that the deviation of the matter overdensity evolution is strong for small scales and weak for large scales, while it is negligible for large redshifts.

I. INTRODUCTION

Cosmological data from a wide range of sources including type Ia supernovae [1–3], the cosmic microwave background [4], baryon acoustic oscillations [5, 6], cluster gas fractions [7, 8] and gamma ray bursts [9, 10] seem to indicate that at least 70% of the energy density in the universe is in the form of an exotic, negative-pressure component, which drives the universe acceleration. Although the simplest way to explain this behavior is the consideration of a cosmological constant [11], the known fine-tuning problem led to the dark energy paradigm. The dynamical nature of dark energy, at least in an effective level, can originate from a variable cosmological "constant" [12], or from various fields, such is a canonical scalar field (quintessence) [13–17], a phantom field, that is a scalar field with a negative sign of the kinetic term [18], the combination of quintessence and phantom in a unified model named quintom [19], or from k-essence [20] and unparticles [21]. On the other hand, the dynamical dark energy can effectively be described by modifying gravity itself, using a functions of the curvature scalar [22], of the Gauss-Bonnet invariant [23], or of the square of the Weyl tensor [24], higher derivatives in the action [25], braneworld extensions [26], string-inspired constructions [27], holographic properties [28], UV modifications [29] etc.

An interesting alternative to General Relativity is the so-called $f(T)$ gravity, which has recently received considerable attention as a possible explanation of the late-time acceleration [30–32]. It is based on the old idea of the "teleparallel" equivalence of General Relativity (TEGR) [33, 34], which, instead of using the Reimann-Cartan space with curvature but no torsion defined via the Levi-Civita connection, uses the Weitzenböck connection that has no curvature but only torsion. The dynamical objects in such a framework are the four linearly independent vierbeins. The advantage of this framework is that the torsion tensor is formed solely from products of first derivatives of the tetrad. As described in [34], the Lagrangian density, T, can then be constructed from this torsion tensor under the assumptions of invariance under general coordinate transformations, global Lorentz transformations, and the parity operation, along with requiring the Lagrangian density to be second order in the torsion tensor. However, instead of using the tensor scalar T the authors of [31, 32] generalized the above formalism to a modified $f(T)$ version, thus making the Lagrangian density a function of T, similar to the well-known extension of $f(R)$ Einstein-Hilbert action. In comparison with $f(R)$ gravity, whose fourth-order equations may lead to pathologies, $f(T)$ gravity has the significant advantage of producing field equations which are at most second order in field derivatives. This feature has led to a rapidly increasing interest in the literature, and apart from obtaining acceleration [31, 32] one can reconstruct a variety of cosmological evolutions [35, 36], add a scalar field [37], examine the conformal transformations [38], use observational data in order to constrain the model parameters [39], examine the dynamical behavior of the scenario [40] and the possibility of the phantom divide crossing [41, 42], and proceed beyond the background evolution, investigating the vacuum and matter perturbations [43].

In this paper we are interested in constructing $f(T)$ scenarios, that exhibit the same background behavior with any given cosmological model. However, we additionally investigate the perturbation evolution, and in particular we examine the growth of matter overdensity, since it distinguishes between the dynamical dark energy scenario and its equivalent family of $f(T)$ models.

The layout of this paper is as follows. In Section II we briefly review the cosmology of $f(T)$ gravity, both at the background and linearized regimes. In Section III we present the general formalism of reconstructing

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the equivalent $f(T)$ models of any given dynamical dark energy scenario, as well as the formalism of using perturbations in order to distinguish them. In Section IV we apply the obtained formalism in the quintessence scenario. Finally, Section V summarizes our results.

II. $f(T)$ GRAVITY AND COSMOLOGY

In this section we present $f(T)$ gravity, we provide the background cosmological equations in a universe governed by such a modified gravitational sector, and we give the first order perturbed equations. Throughout the work we consider a flat Friedmann-Robertson-Walker (FRW) background geometry with metric

$$ds^2 = dt^2 - a^2(t)\,\delta_{ij}dx^i dx^j, \qquad (1)$$

where $a(t)$ is the scale factor. In this manuscript our notation is as follows: Greek indices $\mu, \nu,...$ run over all coordinate space-time 0, 1, 2, 3, lower case Latin indices (from the middle of the alphabet) $i, j, ...$ run over spatial coordinates 1, 2, 3, capital Latin indices $A, B,...$ run over the tangent space-time 0, 1, 2, 3, and lower case Latin indices (from the beginning of the alphabet) $a, b,...$ will run over the tangent space spatial coordinates 1, 2, 3.

A. $f(T)$ gravity

Let us present $f(T)$ gravity. As stated in the Introduction, the dynamical variable of the old "teleparallel" gravity, as well as its $f(T)$ extension, is the vierbein field $\mathbf{e}_A(x^\mu)$, where capital as well as Greek indices take the values $0, 1, 2, 3$. This forms an orthonormal basis for the tangent space at each point x^μ of the manifold, that is $\mathbf{e}_A \cdot \mathbf{e}_B = \eta_{AB}$, where $\eta_{AB} = diag(1, -1, -1, -1)$. Furthermore, the vector $\mathbf{e}_A$ can be analyzed with the use of its components e_A^μ in a coordinate basis, that is $\mathbf{e}_A = e_A^\mu \partial_\mu$.

In such a framework, the metric tensor is obtained from the dual vierbein as

$$g_{\mu\nu}(x) = \eta_{AB}\, e_\mu^A(x)\, e_\nu^B(x). \qquad (2)$$

Contrary to General Relativity, which uses the torsionless Levi-Civita connection, in the present formalism ones uses the curvatureless Weitzenböck connection [44], whose torsion tensor reads

$$T^\lambda_{\ \mu\nu} = \overset{\mathbf{w}}{\Gamma}^\lambda_{\ \nu\mu} - \overset{\mathbf{w}}{\Gamma}^\lambda_{\ \mu\nu} = e_A^\lambda\,(\partial_\mu e_\nu^A - \partial_\nu e_\mu^A). \qquad (3)$$

Moreover, the contorsion tensor, which equals the difference between Weitzenböck and Levi-Civita connections, is defined as

$$K^{\mu\nu}_{\ \ \rho} = -\frac{1}{2}\Big(T^{\mu\nu}_{\ \ \rho} - T^{\nu\mu}_{\ \ \rho} - T_\rho^{\ \mu\nu}\Big). \qquad (4)$$

Finally, it proves useful to define

$$S_\rho^{\ \mu\nu} = \frac{1}{2}\Big(K^{\mu\nu}_{\ \ \rho} + \delta_\rho^\mu\, T^{\alpha\nu}_{\ \ \alpha} - \delta_\rho^\nu\, T^{\alpha\mu}_{\ \ \alpha}\Big). \qquad (5)$$

Note the antisymmetric relations $T^\lambda_{\ \mu\nu} = -T^\lambda_{\ \nu\mu}$ and $S_\rho^{\ \mu\nu} = -S_\rho^{\ \nu\mu}$, as can be easily verified. Using these quantities one can define the so called "teleparallel Lagrangian" as [34, 45, 46]

$$L_T \equiv S_\rho^{\ \mu\nu}\, T^\rho_{\ \mu\nu}. \qquad (6)$$

In summary, in the present formalism, all the information concerning the gravitational field is included in the torsion tensor $T^\lambda_{\ \mu\nu}$, and the teleparallel Lagrangian L_T gives rise to the dynamical equations for the vierbein, which imply the Einstein equations for the metric.

From the above discussion one can deduce that the teleparallel Lagrangian arises from the torsion tensor, similar to the way the curvature scalar arises from the curvature (Riemann) tensor. Thus, one can simplify the notation by replacing the symbol L_T by the symbol T, which is the torsion scalar [32].

While in teleparallel gravity the action is constructed by the teleparallel Lagrangian $L_T = T$, the idea of $f(T)$ gravity is to generalize T to a function $T + f(T)$, which is similar in spirit to the generalization of the Ricci scalar R in the Einstein-Hilbert action to a function $f(R)$. In particular, the action in a universe governed by $f(T)$ gravity reads:

$$I = \frac{1}{16\pi G}\int d^4x e\,[T + f(T) + L_m], \qquad (7)$$

where $e = \det(e_\mu^A) = \sqrt{-g}$ and L_m stands for the matter Lagrangian. We mention here that since the Ricci scalar R and the torsion scalar T differ only by a total derivative [47], in the case where $f(T)$ is a constant (which will play the role of a cosmological constant) the action (7) is equivalent to General Relativity with a cosmological constant.

Finally, we mention that throughout this work we use the common choice for the form of the vierbien, namely

$$e_\mu^A = \mathrm{diag}(1, a, a, a). \qquad (8)$$

It can be easily found that the family of vierbiens related to (8) through global Lorentz transformations, lead to the same equations of motion. Note however that, as it was shown in [48], $f(T)$ gravity does not preserve local Lorentz invariance. Thus, one should in principle study the cosmological consequences of a more general vierbien ansatz. Definitely the subject needs further investigation and it is left for a future work.

B. Background $f(T)$ cosmology

Let us now present the background cosmological equations in a universe governed by $f(T)$ gravity. Variation of the action (7) with respect to the vierbein gives the equations of motion

$$e^{-1}\partial_\mu(eS_A{}^{\mu\nu})[1+f'(T)] - e_A^\lambda T^\rho{}_{\mu\lambda}S_\rho{}^{\nu\mu}[1+f'(T)] + S_A{}^{\mu\nu}\partial_\mu(T)f''(T) - \frac{1}{4}e_A^\nu[T+f(T)] = 4\pi G e_A^\rho \overset{\text{em}}{T}{}_\rho{}^\nu, \qquad (9)$$

where a prime denotes the derivative with respect to T and the mixed indices are used as in $S_A{}^{\mu\nu} = e_A^\rho S_\rho{}^{\mu\nu}$. Note that the tensor $\overset{\text{em}}{T}{}_\rho{}^\nu$ on the right-hand side is the usual energy-momentum tensor, in which we have added an overset label in order to avoid confusion with the torsion tensor.

If we assume the background to be a perfect fluid, then the energy momentum tensor takes the form

$$\overset{\text{em}}{T}{}_{\mu\nu} = pg_{\mu\nu} - (\rho+p)u_\mu u_\nu, \qquad (10)$$

where u^μ is the fluid four-velocity. Note that we are following the conventions of [47], but with an opposite signature metric. Thus, one can see that the equations (9) lead to the background (Friedmann) equations

$$H^2 = \frac{8\pi G}{3}\rho_m - \frac{f(T)}{6} - 2f'(T)H^2 \qquad (11)$$

$$\dot{H} = -\frac{4\pi G(\rho_m + p_m)}{1 + f'(T) - 12H^2 f''(T)}. \qquad (12)$$

In these expressions we have introduced the Hubble parameter $H \equiv \dot{a}/a$, where a dot denotes a derivative with respect to coordinate time t. Moreover, ρ_m and p_m stand respectively for the energy density and pressure of the matter content of the universe, with equation-of-state parameter $w_m = p_m/\rho_m$. Finally, we have employed the very useful relation

$$T = -6H^2, \qquad (13)$$

which straightforwardly arises from evaluation of (6) for the unperturbed metric.

Observing the form of the first Friedmann equation (11), and comparing to the standard form, we deduce that the second and third terms on the right hand side constitute effectively the dark energy sector, which in general presents a dynamical behavior. In particular, one can define the dynamical dark energy (DDE) density as [32]:

$$\rho_{\text{DDE}} \equiv \frac{3}{8\pi G}\left[-\frac{f(T)}{6} - 2f'(T)H^2\right], \qquad (14)$$

while its equation-of-state parameter reads:

$$w = -\frac{f/T - f'(T) + 2Tf''(T)}{[1 + f'(T) + 2Tf''(T)][f/T - 2f'(T)]}. \qquad (15)$$

Thus, in principle, any dynamical dark energy scenario, with a given ρ_{DDE} or a given w, has its $f(T)$ equivalent, and the corresponding reconstruction procedure will be described in section III. Lastly, note that General Relativity is recovered by setting $f(T)$ to a constant (which will play the role of a cosmological constant), as expected.

C. Linear Perturbations in $f(T)$ gravity

We now recall the first order perturbations of $f(T)$ gravity. We refer the reader to [43] for full details of the calculation, summarizing only the relevant results in this section. The perturbed metric in Newtonian gauge with signature $(+ - - -)$ is written as

$$ds^2 = (1+2\psi)dt^2 - a^2(1-2\phi)\delta_{ij}dx^i dx^j, \qquad (16)$$

where the scalars ϕ and ψ are functions of $\mathbf{x}$ and t. Writing the vierbein as

$$e_\mu^A = \bar{e}_\mu^A + t_\mu^A, \qquad (17)$$

where $\bar{e}_\mu^A$ is the unperturbed vierbein, for the above metric we obtain

$$\bar{e}_\mu^0 = \delta_\mu^0 \quad \bar{e}_\mu^a = \delta_\mu^a a \quad \bar{e}_0^\mu = \delta_0^\mu \quad \bar{e}_a^\mu = \frac{\delta_a^\mu}{a} \qquad (18)$$

$$t_\mu^0 = \delta_\mu^0\psi \quad t_\mu^a = -\delta_\mu^a a\phi \quad t_0^\mu = -\delta_0^\mu\psi \quad t_a^\mu = \frac{\delta_a^\mu}{a}\phi, \ (19)$$

with indicial notation as stated at the beginning of section II. Moreover, unless otherwise indicated, subscripts zero and one will generally denote respectively zeroth and linear order values of quantities. We have also incorporated the additional simplifying assumption that the perturbations t_μ^A are diagonal. With these perturbations the determinant becomes

$$e = \det(e_\mu^A) = a^3(1 + \psi - 3\phi). \qquad (20)$$

The perturbations of the energy-momentum tensor are expressed as [43]

$$\delta\overset{\text{em}}{T}{}_0^0 = -\delta\rho_m \qquad (21)$$

$$\delta\overset{\text{em}}{T}{}_0^a = a^{-2}(\rho_m + p_m)(-\partial_a\delta u) \qquad (22)$$

$$\delta\overset{\text{em}}{T}{}_a^0 = (\rho_m + p_m)(\partial_a\delta u) \qquad (23)$$

$$\delta\overset{\text{em}}{T}{}_a^b = \delta_{ab}\delta p. \qquad (24)$$

Pressureless matter implies that $\phi = \psi$, a relation that simplifies the calculations significantly. Additionally, it proves convenient to transform to Fourier space, where any first order quantity y is expanded as

$$y(t, \mathbf{x}) = \int \frac{d^3k}{(2\pi)^{\frac{3}{2}}} y_k(t)e^{i\mathbf{k}\cdot\mathbf{x}}. \qquad (25)$$

In what follows, the k-subscripts in a quantity denotes its Fourier transformation.

Finally, from the above one can result to the following evolution equation for the metric perturbation [43]:

$$\ddot{\phi}_k + \Gamma(f)\dot{\phi}_k + \omega^2(f)\phi_k = 0, \qquad (26)$$

where ω^2 and Γ are respectively given by

$$\omega^2(f) = \frac{\frac{3H^2}{2} + \dot{H} - \frac{f}{4} + \dot{H}f' - \left(36H^4 - 48H^2\dot{H}\right)f'' + 144H^4\dot{H}f'''}{1 + f' - 12H^2f''} \tag{27}$$

$$\Gamma(f) = \frac{4H\left[1 + f' - \left(12H^2 + 9\dot{H}\right)f'' + 36H^2\dot{H}f'''\right]}{1 + f' - 12H^2f''}, \tag{28}$$

with primes denoting derivatives with respect to T.

D. Growth of perturbations

In order to study the growth of perturbations in a matter-only universe ($p_m = \delta p_m = 0$) we define as usual the overdensity δ as

$$\delta \equiv \frac{\delta\rho_m}{\rho_m}. \tag{29}$$

Thus, the relativistic version of the Poisson equation in $f(T)$ gravity reads [43]

$$3H\left(1 + f' - 12H^2f''\right)\dot{\phi}_k$$
$$+ \left[\left(3H^2 + k^2/a^2\right)\left(1 + f'\right) - 36H^4f''\right]\phi_k$$
$$+ 4\pi G\rho\,\delta_k = 0. \tag{30}$$

Equations (26) and (30) can be used to evolve the matter overdensity for a given $f(T)$ model.

III. DYNAMICAL DARK ENERGY AND ITS $f(T)$ EQUIVALENT

In this work we are interested in constructing the $f(T)$ scenario that effectively leads to the same background behavior with a given cosmological model. We stress that this procedure has a full generality, that is for any dynamical dark energy (DDE) scenario we can construct its $f(T)$ equivalent, as long as we know the evolution of the dark energy density ρ_{DDE}. However, despite the coincidence of the background behavior, the perturbations can be used to distinguish the given DDE model from the corresponding $f(T)$ reconstruction.

A. Reconstructing the corresponding $f(T)$ family for any given Dynamical Dark Energy

In order to perform the aforementioned reconstruction at the background level we start from the ρ_{DDE} effective definition (14). Using the fact that $\dot{f} = -12f'H\dot{H}$ (since $T = -6H^2$), we can extract a differential equation for f

in terms of ρ_{DDE}, namely:

$$\dot{f} = \frac{\dot{H}}{H}\left(f + 16\pi G\rho_{\text{DDE}}\right) \tag{31}$$

The solution to this first-order differential equation can be written in closed form as

$$f(H) = 16\pi GH \int \frac{\rho_{\text{DDE}}}{H^2}dH + CH, \tag{32}$$

where C is an integration constant, and the corresponding $f(T)$ can be straightforwardly obtained substituting H by $\sqrt{-T/6}$. In summary, relation (32) describes a one-parameter family of solutions, characterized by the parameter C, which by construction mimics perfectly the background evolution of the given dynamical dark energy density. A self-consistency test for this is that if one use the above $f(T)$ and the given H to calculate the equation-of-state parameter for the $f(T)$ equivalent through (15), he finds exactly the same result as using the conservation equation $\dot{\rho}_{DDE} = -3H\rho_{DDE}(1 + w)$ to calculate the equation-of-state parameter for the given DDE model. Finally, for the special case of General Relativity with a cosmological constant (Λ) (where $\rho_{\text{DDE}} = \rho_\Lambda \equiv \Lambda/(8\pi G)$), with $C = 0$, the corresponding $f(T)$ model reduces to the expected $f = -2\Lambda$.

Let us now make a comment on the parameter C. Interestingly, and as expected, the C term in (32), which corresponds to a term proportional to $(-T)^{1/2}$ in $f(T)$, does not have any effect on the background dynamics, since it disappears from both Friedmann equations (11) and (12), and consequently from any other background-level quantity such as luminosity distances or the equation of state. In other words, the various specific scenarios of the family of solutions given by (32), are indistinguishable from each other at the background level.

B. Using perturbations to distinguish between Dynamical Dark Energy and the corresponding f(T) family

In the previous subsection, we showed that the DDE and the corresponding $f(T)$ models are perfectly indistinguishable at the background level. However, this is not anymore the case if one takes into account the perturbations, as we show in this subsection. In particular,

the perturbations can break the above degeneracy, that is the growth history is different for the DDE as well as for each member of the corresponding reconstructed $f(T)$ family.

Let us now proceed to the investigation of the perturbations. For convenience, in the following we express the dimensional constant C of relation (32) in terms of the dimensionless quantity

$$C_M \equiv -\frac{CH_i}{16\pi G\rho_{\mathrm{DDE},i}}, \tag{33}$$

where H_i is the Hubble parameter at an initial redshift z_i, that is $H_i \equiv H(z=z_i)$, where we use the redshift z as the independent variable instead of the scale factor ($1+z = a_0/a$ with a_0 the present scale-factor value). $\rho_{\mathrm{DDE},i}$ is the initial energy density of the DDE. If we assume $\rho_{\mathrm{DDE},i}$ to be slowly varying during matter domination, then the value of the integral in (32) at the initial time ($z=z_i$) can be approximated by $-16\pi G\rho_{\mathrm{DDE},i}$, and then C_M can be physically interpreted from the fact that the initial value of $f(H)$ at $z=z_i$ becomes $-16\pi G(C_M+1)\rho_{\mathrm{DDE},i}$.

Amongst the individual members of a reconstructed family of $f(T)$ models, corresponding to a given DDE scenario, the growth histories can be different for different choices of C_M. In particular, the deviations in the growth history for different values of C_M are larger for smaller scales, while they diminish for large scales. This behavior can be qualitatively understood from equation (30). Using δ_f to denote the matter perturbations under the reconstructed $f(T)$ scenarios, we perform in (30) the transformation $f(T): f(T) \to F(T) + CH$, (where $F(T)$ is clearly the $C_M = 0$ member of the family). We find that (30) transforms to

$$
\begin{aligned}
&3H\left(1 + F' - 12H^2F''\right)\dot\phi_k \\
&+ \left[\left(3H^2 + k^2/a^2\right)\left(1+F'\right) - 36H^4F''\right]\phi_k \\
&+ 4\pi G\rho_m\,\delta_{fk} = \frac{C}{12H}\frac{k^2}{a^2}\phi_k.
\end{aligned}
\tag{34}
$$

As we observe, the right-hand-side of the above equation indicates the difference between the δ_f of the $C_M = 0$ model and that of the various $C_M \neq 0$ ones. It is therefore clear that for large scales ($k \to 0$) the difference between the $C_M \neq 0$ and the $C_M = 0$ models becomes small. In summary, perturbations indeed uniquely distinguish a DDE scenario from the different members of the corresponding reconstructed $f(T)$ family.

Finally, we mention that our results are similar in spirit to [49] in the context of $f(R)$ gravity, where the authors find that any expansion history of the universe can be replicated by an one-parameter family of $f(R)$ models, characterized by a parameter $B \propto f''(R)$, which provides a variety of different behaviors for the different members of the family. However, in the case of the present work there is a difference, namely the various models of the family have a relatively similar evolution even for large differences in the parameter C_M.

IV. APPLICATION: $f(T)$ EQUIVALENT FOR QUINTESSENCE

In the previous section we described how we can construct $f(T)$ models that exhibit the same behavior at the background level with any given dynamical dark energy (DDE) scenario. Furthermore, we showed how the examination of perturbations can be used in order to distinguish the DDE scenario from the various members of the reconstructed $f(T)$ family, as well as these various members from each other. In the present section, in order to provide specific examples, we apply these procedures in the well-known dynamical dark energy scenario of quintessence.

In the quintessence paradigm, the dark energy sector is attributed to a homogeneous scalar field Φ, while the matter sector is described by a pressureless perfect fluid with energy density ρ_m [13–16]. In the case of flat geometry the Friedman equation reads

$$H^2 = \frac{8\pi G}{3}\left[\rho_m + \frac{\dot\Phi^2}{2} + V(\Phi)\right], \tag{35}$$

where $V(\Phi)$ is the scalar-field potential. In these models, the field typically rolls down a very shallow potential, eventually coming to rest when it can find a local minimum. In particular, the evolution equation for Φ reads:

$$\ddot\Phi + 3H\dot\Phi + V'(\Phi) = 0, \tag{36}$$

while the corresponding one for ρ_m is

$$\dot\rho_m + 3H\rho_m = 0. \tag{37}$$

Additionally, one can define the effective dark energy density and pressure in field terms as

$$\rho_{\mathrm{DDE}} \equiv \frac{\dot\Phi^2}{2} + V(\Phi) \tag{38}$$

$$p_{\mathrm{DDE}} \equiv \frac{\dot\Phi^2}{2} - V(\Phi), \tag{39}$$

and thus the corresponding dark-energy equation-of-state parameter as

$$w_{\mathrm{DDE}} \equiv \frac{p_{\mathrm{DDE}}}{\rho_{\mathrm{DDE}}} = \frac{\frac{\dot\Phi^2}{2} - V(\Phi)}{\frac{\dot\Phi^2}{2} + V(\Phi)}, \tag{40}$$

which obviously present a dynamical behavior in general. Finally, in such a scenario, the growth of perturbations is governed by the linearized Einstein equations (see e.g. [50]):

$$\dot v_f = -\phi_k$$

$$\dot\delta_k = 3\dot\phi_k + \frac{k^2}{a^2}v_f$$

$$\ddot\phi_k + 4H\dot\phi_k = 8\pi G\left(\frac{\dot\Phi^2}{2} - V\right)\phi_k + 4\pi G\left(\dot\Phi\delta\dot\Phi - V'\delta\Phi\right)$$

$$\delta\ddot\Phi = -3H\delta\dot\Phi - \left(\frac{k^2}{a^2} + V''\right)\delta\Phi + 4\dot\Phi\dot\phi_k - 2V'\phi_k, \tag{41}$$

where v_f is the velocity potential of the matter fluid and primes denote derivatives with respect to Φ.

One last subject that has to be settled before proceeding forward is the choice of the quintessence potential $V(\Phi)$. Amongst the various ansatzes of the literature, in this work we focus on three commonly studied cases, namely:

1. The Pseudo-Nambu-Goldstone-Boson (PNGB) Model:

$$V(\Phi) = A^4 \left[1 + \cos(\Phi/f)\right], \qquad (42)$$

where A is the energy scale of the potential, and f is a symmetry restoration scale. This model was first proposed in [51], while its cosmological applications were discussed in [52].

2. The Exponential Model:

$$V(\Phi) = A^4 \exp(-\beta\Phi/M_{\mathrm{Pl}}), \qquad (43)$$

where M_{Pl} is the Planck mass. Exponential potentials arise in a variety of contexts, such as higher dimensional gravitational theories, moduli fields, and also non-perturbative effects such as gaugino condensation (see e.g. [53] and references therein).

3. The Power Law Model:

$$V(\Phi) = A^{4+n}\Phi^{-n}. \qquad (44)$$

Power law potentials have been studied extensively in [13, 14, 54], and have been shown to arise in the context of SUSY in [55].

The usual method of differentiating various types of DDE models is to examine their phase-space behavior, focusing in particular on the dark-energy equation-of-state parameter and its evolution [56]. The various models can either fall into a "thawing" or "freezing" category. The former is thusly named because the field is only recently beginning to exhibit dynamical behavior since it is being frozen at some distance from its minimum due to Hubble friction, while the name of the latter arises form the fact that it is dynamical throughout much of the universe's history but it becomes frozen into place at late times (during the time of dark energy domination). Amongst the three choices above, the first two are thawing models while the third one is a freezing model.

A. Reconstructing $f(T)$ for quintessence

Let us now reconstruct an $f(T)$ cosmology that exhibits the same behavior at the background level with a given quintessence scenario. As described in subsection III A, the corresponding $f(T)$ family of models is given by relation (32), where in the quintessence case the dark energy density ρ_{DDE} is given by (38), while the cosmological equations close by (35)-(37).

In a non-trivial quintessence scenario, (32) cannot be solved analytically. Thus, in the following we perform a numerical elaboration in the case of the three quintessence models described above. In particular, knowing the evolution of ρ_{DDE} and of the effective dark-energy equation-of-state parameter w for the three quintessence models, we numerically reconstruct the corresponding $f(T)$ family using (32), and then we numerically extract the w-behavior using (15).

For the three quintessence potentials we suitably choose the parameter values in order for w to lie within its observational limits $-1 < w_{\mathrm{DDE}} < -0.9$ for low redshifts [4]. We solve the cosmological equations determining the initial conditions at a redshift z_i deep inside the matter dominated era, and we evolve the system until the density parameter of matter $\Omega_m(z) \equiv 8\pi G\rho_m/(3H^2(z))$ becomes equal to ≈ 0.3 at $z = 0$, as it is required by observations. For our numerical analysis, we work in units where $8\pi G = 1$ and $\rho_\Lambda = 4/3$.
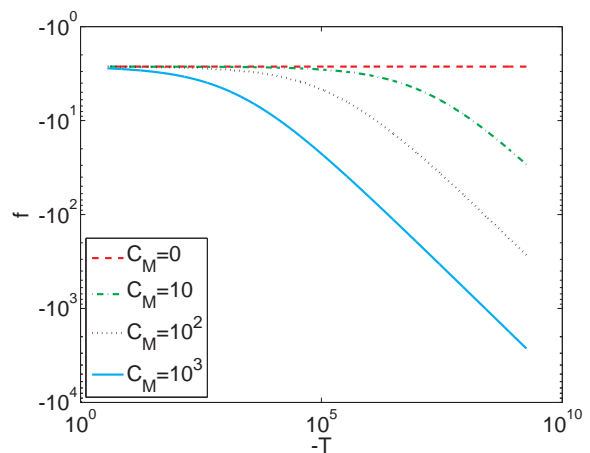

Figure 1: *For members of the reconstructed $f(T)$ family of models for the Pseudo-Nambu-Goldstone-Boson (PNGB) quintessence scenario given by (42), characterized by four choices of the parameter C_M.*

In Figures 1-3 we present the reconstructed $f(T)$ for the three quintessence models, namely the Pseudo-Nambu-Goldstone-Boson (42), the exponential (43), and the power-law (44) models, respectively (such figures are easily created since we know numerically $f(z)$ and $H(z)$, that is $T(z)$, and thus we obtain $f(T)$). In each figure, we depict $f(T)$ for four members of the $f(T)$ family of models, characterized by four choices of the parameter C_M.

We mention that the C-term in (32) behaves like a decaying mode, as it falls of approximately as $a^{-3/2}$ during early times, making the $C = 0$ ($C_M = 0$) case an attractor for this family of models. This is demonstrated in Figures 1-3, where it is clear that the $C_M \neq 0$ models approach the $C_M = 0$ one, for the reconstructions of all three quintessence scenarios. Finally, note that we have focused only on positive C_M-values, since negative val-
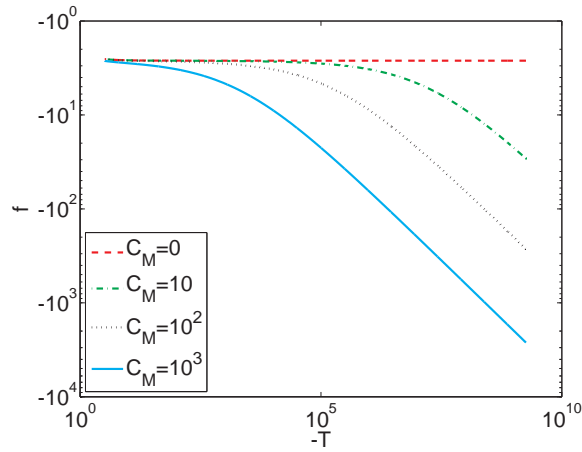
Figure 2: *For members of the reconstructed f(T) family of models for the exponential quintessence scenario given by (43), characterized by four choices of the parameter C_M.*
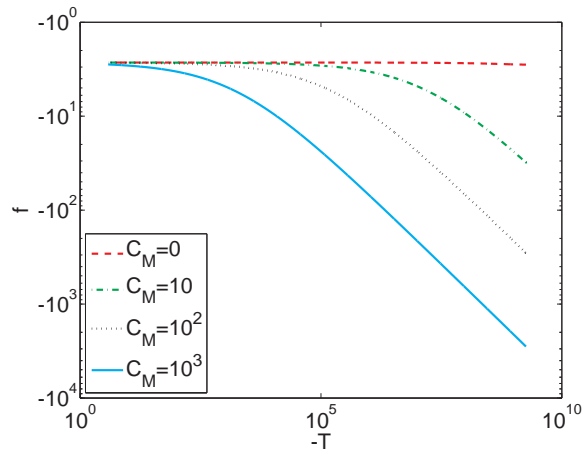

Figure 3: *For members of the reconstructed f(T) family of models for the power-law quintessence scenario given by (44), characterized by four choices of the parameter C_M.*
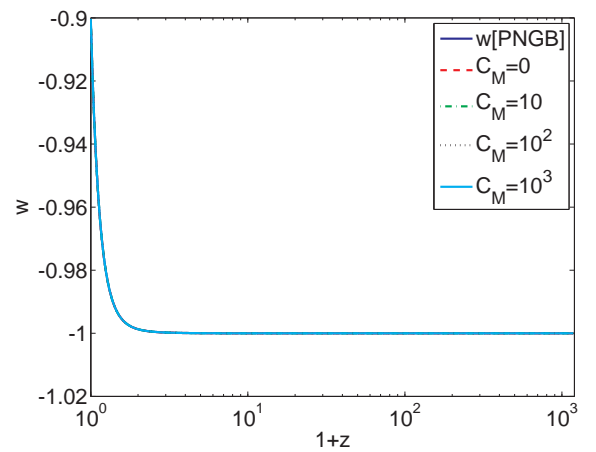

Figure 4: *The evolution of the dark-energy equation-of-state parameter $w(z)$, as a function of the redshift, for the Pseudo-Nambu-Goldstone-Boson (PNGB) quintessence scenario given by (42), as well as for four members of its corresponding reconstructed f(T) family of models (characterized by four choices of the parameter C_M).*
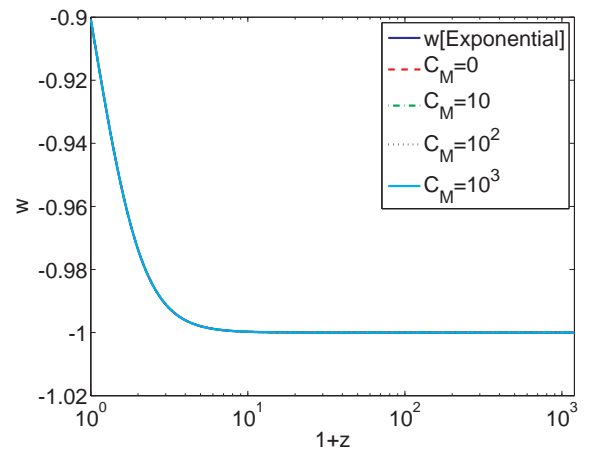

Figure 5: *The evolution of the dark-energy equation-of-state parameter $w(z)$, as a function of the redshift, for the exponential quintessence scenario given by (43), as well as for four members of its corresponding reconstructed f(T) family of models (characterized by four choices of the parameter C_M).*

ues will produce graphs which are symmetrically reflected about the $f(z) = f(z = 0) = -2\Lambda$ line.

Having reconstructed the $f(T)$ family of models that corresponds to the three quintessence scenarios, in Figures 4-6 we present the behavior of $w(z)$, which is a basic observable, for each potential respectively, according to (40). In each figure, we depict additionally the $w(z)$ of the corresponding reconstructed $f(T)$ family of models, according to (15), for four values of the parameter C_M. As we observe, in all cases there is a perfect overlap of the corresponding figures, which verifies the fact that for background-level quantities, such is the dark-energy equation-of-state parameter w, any dynamical dark energy scenario is indistinguishable by construction from its corresponding $f(T)$ equivalent family of models.

B. Using perturbations to distinguish between quintessence and the corresponding f(T) family

In the previous subsection, we applied our general $f(T)$-reconstruction formalism of section III A in the case of quintessence dynamical dark energy scenario. In the present subsection we investigate the perturbations following subsection III B, in order to distinguish between quintessence and its equivalent family of models.

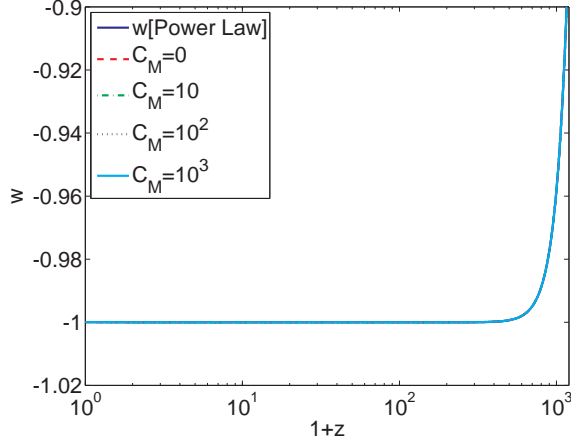The perturbation evolution for quintessence is given by equations (41), while for the family of $f(T)$ models

Figure 6: *The evolution of the dark-energy equation-of-state parameter $w(z)$, as a function of the redshift, for the power-law quintessence scenario given by (44), as well as for four members of its corresponding reconstructed $f(T)$ family of models (characterized by four choices of the parameter C_M).*

by (34). We follow the numerical elaboration described in the background case, and we additionally choose the initial velocity of the metric perturbation to be at zero.
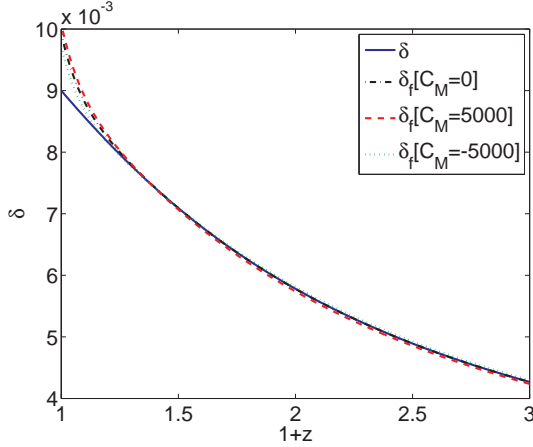

Figure 7: *The evolution of the matter overdensity δ as a function of the redshift z, on a scale of $k = 10^{-3} h\ Mpc^{-1}$, for the PNGB scenario given by (42), as well as for three members of its corresponding reconstructed $f(T)$ family of models (characterized by three choices of the parameter C_M).*

In Fig. 7 we demonstrate the evolution of the matter overdensity δ as a function of the redshift, in the case of Pseudo-Nambu-Goldstone-Boson (PNGB) quintessence scenario given by (42), in a scale of $k = 10^{-3} h\ \mathrm{Mpc}^{-1}$, while in Fig. 8 we provide the same graph but in a scale of $k = 1 h\ \mathrm{Mpc}^{-1}$. In the same Figures, we additionally depict the δ-evolution for three members of the corresponding reconstructed $f(T)$ family, determined by three distinct values of the parameter C_M ($C_M = \pm 5000$ and

$C_M = 0$). Similarly, in Figures 9 and 10 we provide the corresponding plots for the exponential scenario given by (43), while in Figures 11 and 12 we show the corresponding graphs for the power-law scenario given by (44).
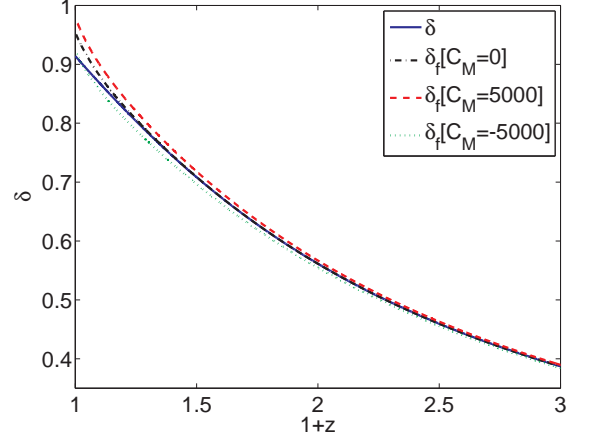

Figure 8: *The evolution of the matter overdensity δ as a function of the redshift z, on a scale of $k = 1 h\ Mpc^{-1}$, for the PNGB scenario given by (42), as well as for three members of its corresponding reconstructed $f(T)$ family of models (characterized by three choices of the parameter C_M).*
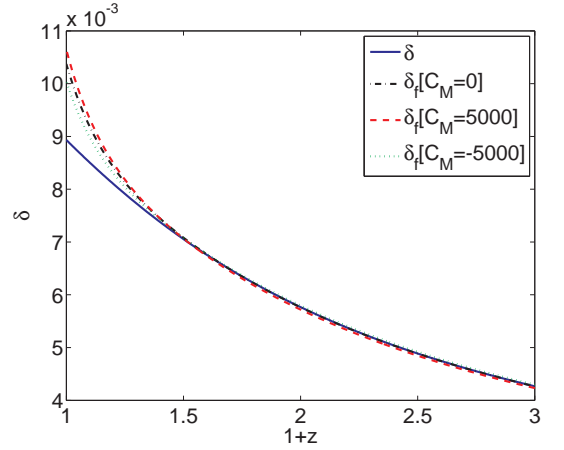

Figure 9: *The evolution of the matter overdensity δ as a function of the redshift z, on a scale of $k = 10^{-3} h\ Mpc^{-1}$, for the exponential scenario given by (43), as well as for four members of its corresponding reconstructed $f(T)$ family of models (characterized by four choices of the parameter C_M).*

From these Figures we observe that the evolution at the perturbation level distinguishes the quintessence scenario from its reconstructed equivalent family of $f(T)$ models, as well as the various $f(T)$ family members from each other. Concerning the scale, as discussed in subsection III B the deviation in δ-evolution is strong for small scales and weak for large scales. Concerning the redshift,
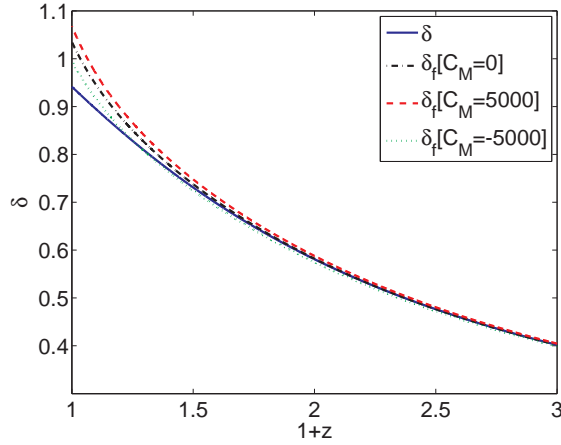
Figure 10: *The evolution of the matter overdensity δ as a function of the redshift z, on a scale of $k = h\ Mpc^{-1}$, for the exponential scenario given by (43), as well as for four members of its corresponding reconstructed $f(T)$ family of models (characterized by four choices of the parameter C_M).*
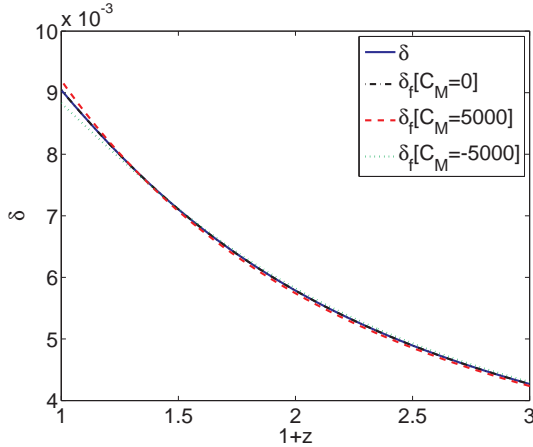

Figure 12: *The evolution of the matter overdensity δ as a function of the redshift z, on a scale of $k = h\ Mpc^{-1}$, for the power-law scenario given by (44), as well as for three members of its corresponding reconstructed $f(T)$ family of models (characterized by three choices of the parameter C_M).*
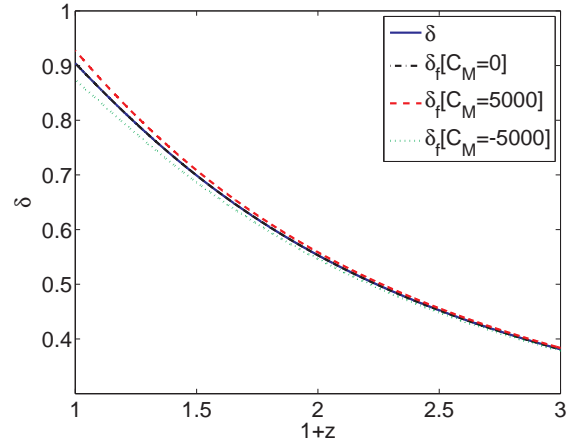

Figure 11: *The evolution of the matter overdensity δ as a function of the redshift z, on a scale of $k = 10^{-3}h\ Mpc^{-1}$, for the power-law scenario given by (44), as well as for three members of its corresponding reconstructed $f(T)$ family of models (characterized by three choices of the parameter C_M).*

for all the models the deviation is negligible for large redshifts $z \gtrsim 3$, which is expected since the universe is matter dominated at that time and the dark-energy sector (DDE or $f(T)$) plays no role in determining the growth of perturbations. Moreover, note that that the shape of the $\delta(z)$-curve differs fundamentally from that of the $\delta_f(z; C_M)$-curves. Therefore, it is possible to choose a value of C_M such that δ and δ_f coincide at a particular redshift, but they will not coincide at other redshifts for this particular choice of C_M.

Finally, as we can see, in general the parameter C_M has a very weak impact on the splitting of δ-evolutions, even for small scales - the difference between the $C_M = 5000$

and the $C_M = -5000$ cases is modest in all the cases considered. This is most likely a result of the attractor nature of the $C_M = 0$ solution described in IV A.

V. CONCLUSIONS

In this work we investigated $f(T)$ cosmology in both the background, as well as in the perturbation level, as a way to mimic the behavior of a given model of dynamical dark energy. $f(T)$ gravity is the extension of the "teleparallel" equivalent of General Relativity, which uses the zero curvature Weitzenböck connection instead of the torsionless Levi-Civita connection, in the same lines as $f(R)$ gravity is the extension of standard General Relativity.

First, we presented the general formalism for reconstructing the $f(T)$ scenario that effectively leads to the same behavior with any given cosmological model, at the background level. As we showed, for a given dynamical dark energy scenario, one can reconstruct its equivalent, one-parameter family of models. Although they exhibit completely indistinguishable background behavior, the perturbations can break the above degeneracy. That is, the growth history is different for the given cosmological scenario as well as for each member of the corresponding reconstructed $f(T)$ family.

In order to present our results more transparently, we applied the aforementioned general formalism in the well known case of quintessence dynamical dark energy scenario. After reconstructing the $f(T)$ family for three quintessence scenarios, we numerically extracted the evolution of the dark-energy equation-of-state parameter, which is a basic observable. Since it is a background quantity it leads to identical behavior for quintessence as

well as for its $f(T)$ equivalent.

However, upon examining the perturbations, and in particular the evolution of the matter overdensity, we do acquire a way to distinguish between quintessence and its $f(T)$ equivalent family, as well as between the infinite members of the reconstructed $f(T)$ family. More specifically, the deviation is strong for small scales and weak for large scales, and additionally it is negligible for large redshifts ($z \gtrsim 3$), since there the universe is matter dominated with a negligible dark-energy sector.

In summary, $f(T)$ gravity can effectively mimic any dynamical dark energy cosmological scenario at the background level. The perturbation analysis can break this degeneracy leading to rejection or acceptance of specific $f(T)$ models. These features make $f(T)$ cosmology an interesting candidate for the description of nature, which requires deeper examination.

[1] M. Kowalski *et al.*, Astrophys. J. **686**, 749 (2008).
[2] L. Perivolaropoulos and A. Shafieloo, Phys. Rev. D **79**, 123502 (2009).
[3] M. Hicken *et al.*, Astrophys. J. **700**, 1097 (2009).
[4] E. Komatsu *et al.*, arXiv:1001.4538 [astro-ph.CO].
[5] D. J. Eisenstein *et al.* [SDSS Collaboration], Astrophys. J. **633**, 560 (2005).
[6] W. J. Percival, S. Cole, D. J. Eisenstein, R. C. Nichol, J. A. Peacock, A. C. Pope and A. S. Szalay, Mon. Not. Roy. Astron. Soc. **381**, 1053 (2007).
[7] L. Samushia, G. Chen and B. Ratra, arXiv:0706.1963 [astro-ph].
[8] S. Ettori *et al.*, arXiv:0904.2740 [astro-ph.CO].
[9] Y. Wang, Phys. Rev. D **78**, 123532 (2008).
[10] L. Samushia and B. Ratra, Astrophys. J. **714**, 1347 (2010).
[11] V. Sahni and A. Starobinsky, Int. J. Mod. Phy. D **9**, 373 (2000); P. J. Peebles and B. Ratra, Rev. Mod. Phys. **75**, 559 (2003).
[12] J. Sola and H. Stefancic, Phys. Lett. B **624**, 147 (2005); I. L. Shapiro and J. Sola, Phys. Lett. B **682**, 105 (2009).
[13] B. Ratra and P. J. E. Peebles, Phys. Rev. D **37**, 3406 (1988).
[14] I. Zlatev, L.-M. Wang, and P.J. Steinhardt, Phys. Rev. Lett. **82**, 896 (1999).
[15] P.J. Steinhardt, L.-M. Wang, and I. Zlatev, Phys. Rev. D **59** 123504 (1999).
[16] L.-M. Wang and P.J. Steinhardt, Astrophys. J. **508** 483 (1998).
[17] C. Wetterich, Nucl. Phys. B **302**, 668 (1988); A. R. Liddle and R. J. Scherrer, Phys. Rev. D **59**, 023509 (1999) S. Dutta and R. J. Scherrer, Phys. Rev. D **78**, 123525 (2008); Z. K. Guo, N. Ohta and Y. Z. Zhang, Mod. Phys. Lett. A **22**, 883 (2007); S. Dutta and R. J. Scherrer, Phys. Rev. D **78**, 083512 (2008); S. Dutta and R. J. Scherrer, Phys. Lett. B **676**, 12 (2009); S. Dutta, E. N. Saridakis and R. J. Scherrer, Phys. Rev. D **79**, 103005 (2009); S. Dutta, S. D. H. Hsu, D. Reeb and R. J. Scherrer, Phys. Rev. D **79**, 103504 (2009); T. Chiba, S. Dutta and R. J. Scherrer, Phys. Rev. D **80**, 043517 (2009); E. N. Saridakis and S. V. Sushkov, Phys. Rev. D **81**, 083510 (2010).
[18] R. R. Caldwell, Phys. Lett. B **545**, 23 (2002); R. R. Caldwell, M. Kamionkowski and N. N. Weinberg, Phys. Rev. Lett. **91**, 071301 (2003); S. Nojiri and S. D. Odintsov, Phys. Lett. B **562**, 147 (2003); Z.-K. Guo, Y.-S. Piao, and Y.-Z. Zhang, Phys. Lett. B **594**, 247 (2004); E. Elizalde, S. Nojiri, and S.D Odintsov, Phys. Rev. D **70**, 043539 (2004); V. K. Onemli and R. P. Woodard, Phys. Rev. D **70**, 107301 (2004); V. Faraoni, Class. Quant. Grav.

22, 3235 (2005); J. Kujat, R.J. Scherrer, and A.A. Sen, Phys. Rev. D **74**, 083501 (2006); T. Chiba, Phys. Rev. D **73**, 063501 (2006); E. N. Saridakis, Phys. Lett. B **676**, 7 (2009); M. R. Setare and E. N. Saridakis, JCAP **0903**, 002 (2009); E. N. Saridakis, Nucl. Phys. B **819**, 116 (2009).
[19] B. Feng, X. L. Wang and X. M. Zhang, Phys. Lett. B **607**, 35 (2005); Z. K. Guo, *et al.*, Phys. Lett. B **608**, 177 (2005); M.-Z Li, B. Feng, X.-M Zhang, JCAP, 0512, 002 (2005); B. Feng, M. Li, Y.-S. Piao and X. Zhang, Phys. Lett. B **634**, 101 (2006); W. Zhao and Y. Zhang, Phys. Rev. D **73**, 123509 (2006); M. R. Setare and E. N. Saridakis, Phys. Lett. B **668**, 177 (2008); M. R. Setare and E. N. Saridakis, JCAP **0809**, 026 (2008); E. N. Saridakis and J. M. Weller, Phys. Rev. D **81**, 123523 (2010); Y. F. Cai, E. N. Saridakis, M. R. Setare and J. Q. Xia, Phys. Rept. **493**, 1 (2010).
[20] T. Chiba, T. Okabe and M. Yamaguchi, Phys. Rev. D **62**, 023511 (2000); C. Armendariz-Picon, V. F. Mukhanov and P. J. Steinhardt, Phys. Rev. Lett. **85**, 4438 (2000); C. Armendariz-Picon, V. F. Mukhanov and P. J. Steinhardt, Phys. Rev. Lett. **85**, 4438 (2000); T. Chiba, Phys. Rev. D **66**, 063514 (2002).
[21] D. C. Dai, S. Dutta and D. Stojkovic, Phys. Rev. D **80**, 063522 (2009).
[22] S. Capozziello, S. Carloni and A. Troisi, Recent Res. Dev. Astron. Astrophys. **1**, 625 (2003); S. M. Carroll, V. Duvvuri, M. Trodden and M. S. Turner, Phys. Rev. D **70**, 043528 (2004); S. Nojiri and S. D. Odintsov, "Modified gravity with negative and positive powers of the curvature: Phys. Rev. D **68**, 123512 (2003); A. De Felice and S. Tsujikawa, arXiv:1002.4928 [gr-qc].
[23] S. Nojiri, S. D. Odintsov and M. Sasaki, Phys. Rev. D **71**, 123509 (2005).
[24] P. D. Mannheim, Prog. Part. Nucl. Phys. **56**, 340 (2006).
[25] S. Nojiri and S. D. Odintsov, Phys. Lett. B **631**, 1 (2005); S. Nojiri and S. D. Odintsov, J. Phys. Conf. Ser. **66**, 012005 (2007).
[26] P. Binétruy, C. Deffayet, D. Langlois, Nucl. Phys. B **565**, 269 (2000); R.G. Cai, Y.G. Gong, B. Wang, JCAP **0603**, 006 (2006); Y.G. Gong, A. Wang, Class. Quantum Grav. **23**, 3419 (2006); F. K. Diakonos and E. N. Saridakis, JCAP **0902**, 030 (2009).
[27] S. Tsujikawa, arXiv:1004.1493 [astro-ph.CO].
[28] S. D. H. Hsu, Phys. Lett. B **594**, 13 (2004); M. Li, Phys. Lett. B **603**, 1 (2004); E. N. Saridakis, Phys. Lett. B **660**, 138 (2008).
[29] P. Horava, Phys. Rev. D **79**, 084008 (2009); G. Calcagni, JHEP **0909**, 112 (2009); E. Kiritsis and G. Kofinas, Nucl. Phys. B **821**, 467 (2009); H. Lu, J. Mei and

C. N. Pope, Phys. Rev. Lett. **103**, 091301 (2009); E. N. Saridakis, Eur. Phys. J. C **67**, 229 (2010); X. Gao, Y. Wang, R. Brandenberger and A. Riotto, Phys. Rev. D **81**, 083508 (2010); G. Leon and E. N. Saridakis, JCAP **0911**, 006 (2009); M. i. Park, JHEP **0909**, 123 (2009); S. Dutta and E. N. Saridakis, JCAP **1001**, 013 (2010); C. Germani, A. Kehagias and K. Sfetsos, JHEP **0909**, 060 (2009); E. Kiritsis, Phys. Rev. D **81**, 044009 (2010); D. Capasso and A. P. Polychronakos, JHEP **1002**, 068 (2010); S. Dutta and E. N. Saridakis, JCAP **1005**, 013 (2010); G. Koutsoumbas and P. Pasipoularides, Phys. Rev. D **82**, 044046 (2010); M. Eune and W. Kim, arXiv:1007.1824 [hep-th]; H. B. Kim and Y. Kim, arXiv:1009.1201 [hep-th]; H. Kasari and T. T. Fujishiro, arXiv:1009.1703 [hep-th]; T. Harko, Z. Kovacs and F. S. N. Lobo, arXiv:1009.1958 [gr-qc];

[30] R. Ferraro and F. Fiorini, Phys. Rev. D **75**, 084031 (2007).

[31] G. R. Bengochea and R. Ferraro, Phys. Rev. D **79**, 124019 (2009).

[32] E. V. Linder, Phys. Rev. D **81**, 127301 (2010).

[33] A. Einstein 1928, Sitz. Preuss. Akad. Wiss. p. 217; ibid p. 224; A. Unzicker and T. Case, *Translation of Einstein's attempt of a unified field theory with teleparallelism* arXiv:physics/0503046.

[34] K. Hayashi and T. Shirafuji, Phys. Rev. D **19**, 3524 (1979); Addendum-ibid. **24**, 3312 (1982).

[35] R. Myrzakulov, arXiv:1006.1120 [gr-qc].

[36] R. Myrzakulov, arXiv:1008.4486 [astro-ph.CO].

[37] K. K. Yerzhanov, S. R. Myrzakul, I. I. Kulnazarov and R. Myrzakulov, arXiv:1006.3879 [gr-qc].

[38] R. J. Yang, arXiv:1010.1376 [gr-qc].

[39] P. Wu and H. Yu, arXiv:1007.2348 [astro-ph.CO].

[40] P. Wu and H. Yu, arXiv:1006.0674 [gr-qc].

[41] P. Wu and H. W. Yu, arXiv:1008.3669 [gr-qc]; K. Karami and A. Abdolmaleki, arXiv:1009.2459 [gr-qc].

[42] K. Bamba, C. Q. Geng and C. C. Lee, arXiv:1008.4036 [astro-ph.CO].

[43] J. B. Dent, S. Dutta and E. N. Saridakis, arXiv:1008.1250 [astro-ph.CO].

[44] Weitzenböck R., *Invarianten Theorie*, (Nordhoff, Groningen, 1923).

[45] J. W. Maluf, J. Math. Phys. **35** (1994) 335.

[46] H. I. Arcos and J. G. Pereira, Int. J. Mod. Phys. D **13**, 2193 (2004).

[47] S. Weinberg, *Cosmology*, Oxford University Press Inc., New York, 2008.

[48] B. Li, T.P. Sotiriou, and J.D. Barrow, arXiv:1010.1041 [gr-qc]

[49] Y. S. Song, W. Hu and I. Sawicki, Phys. Rev. D **75**, 044004 (2007).

[50] S. Dutta and I. Maor, Phys. Rev. D **75**, 063507 (2007); J. B. Dent, S. Dutta and T. J. Weiler, Phys. Rev. D **79**, 023502 (2009); J. B. Dent and S. Dutta, Phys. Rev. D **79**, 063516 (2009).

[51] J.A. Frieman, C.T. Hill, A. Stebbins, and I. Waga, Phys. Rev. Lett. **75**, 2077 (1995).

[52] K. Dutta and L. Sorbo, Phys. Rev. D **75**, 063514 (2007); A. Abrahamse, A. Albrecht, M. Barnard, and B. Bozek, Phys. Rev. D **77**, 103503 (2008); R. de Putter and E. V. Linder, JCAP **0810**, 042 (2008).

[53] E. J. Copeland, A. R. Liddle and D. Wands, Phys. Rev. D **57**, 4686 (1998); A. Albrecht and C. Skordis, Phys. Rev. Lett. **84**, 2076 (2000); T. Barreiro, E. J. Copeland and N. J. Nunes, Phys. Rev. D **61**, 127301 (2000).

[54] A. R. Liddle and R. J. Scherrer, Phys. Rev. D **59**, 023509 (1999); E. N. Saridakis, Nucl. Phys. B **830**, 374 (2010).

[55] P. Binetruy, Phys. Rev. D **60**, 063502 (1999); A. Masiero, M. Pietroni and F. Rosati, Phys. Rev. D **61**, 023504 (2000).

[56] R. Caldwell and E.V. Linder, Phys. Rev. Lett. **95** 141301 (2005)